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Building an Attitude and Orbit Propagator to Simulate Magnetorquer-Based CubeSat Control Systems

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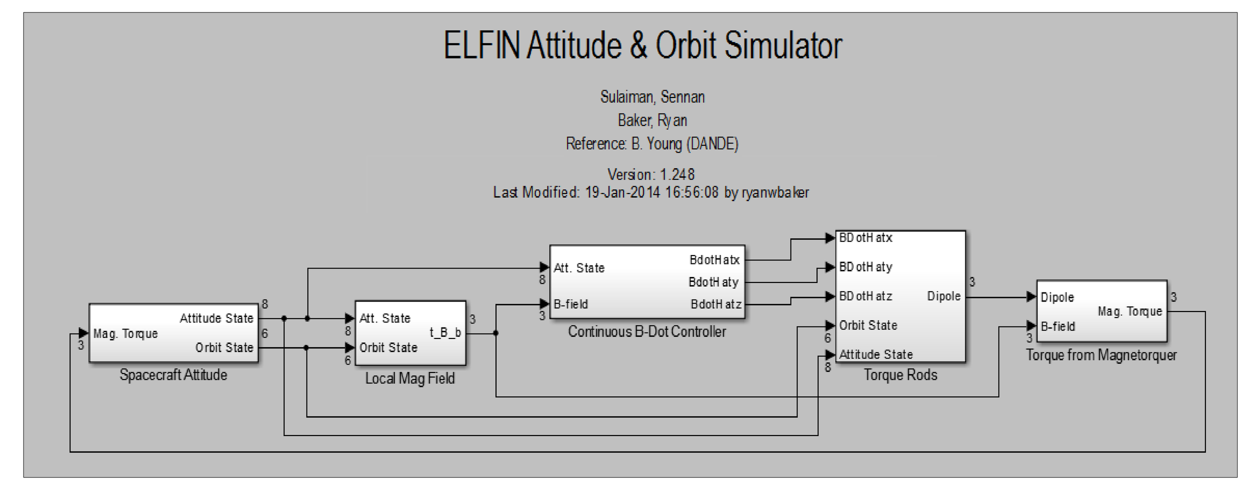
ELFIN (Electron Losses and Field Investigation)

Overview

CubeSats are a form of miniaturized satellite with simplified infrastructure, standardized by California Polytechnic State University and Stanford University with the intent to facilitate the design and flight of an increasing number of missions in a cost-effective fashion. As development of CubeSats has increased for over a decade, many missions are flying increasingly complex systems with payloads capable of cutting-edge science. Development of these increasingly prevalent CubeSats requires accurate and efficient software simulation tools, particularly those related to attitude control and determination where it can be difficult to create flight-like physical test simulations. The ELFIN CubeSat team at UCLA has worked to develop an attitude and orbit propagator specifically for CubeSats with control based on torquing against Earth's magnetic field, enabling tests of these control systems to be carried out within the context of a software simulation.



Geomagnetic Model in Simulation

The IGRF model provides the geomagnetic field at a given point in orbit in the Earth-Centered, Earth-Fixed frame of reference; therefore, a transformation between ECI and ECF (using the Julian day as an argument) is used and the propagated orbit state, in ECI, can therefore be used as the input to this IGRF "lookup table" in the simulation. Finally, the propagated attitude state provides the quaternions necessary to rotate the magnetic field vector from ECI to the body frame of reference, as would be sensed on board the satellite.

Propagating Orbit State

In ECI, the equations for gravity-perturbed motion (Sharaf) are given by the equations to the right, where *x*, *y*, and *z* are the three components of the *R* vector (satellite position in ECI), μ is the standard gravitational parameter of earth, *r* is the norm of the *R* vector, and *c* is given by $c = J_2 \mu R^2_{earth}/2$. J_2 is a constant known as the second zonal harmonic, which accounts for the oblateness of the earth.

$$\ddot{x} = \frac{\partial V}{\partial x} = -\frac{\mu x}{r^3} + 3c\left(\frac{x}{r^5}\right)\left(1 - \frac{5z^2}{r^2}\right),$$

$$\ddot{y} = \frac{\partial V}{\partial y} = -\frac{\mu y}{r^3} + 3c\left(\frac{y}{r^5}\right)\left(1 - \frac{5z^2}{r^2}\right),$$

$$\ddot{z} = \frac{\partial V}{\partial z} = -\frac{\mu z}{r^3} + 3c\left(\frac{z}{r^5}\right)\left(3 - \frac{5z^2}{r^2}\right).$$

Simulating Drag

The effect of atmospheric drag on the acceleration of the satellite must also be taken into account. Acceleration due to drag is given by the following equation, where ρ_a is the atmospheric density, v is the satellite velocity (relative to the atmosphere), A is the cross-sectional area of the satellite projected onto the velocity vector, m is the mass of the satellite, and C_d is the coefficient of drag:

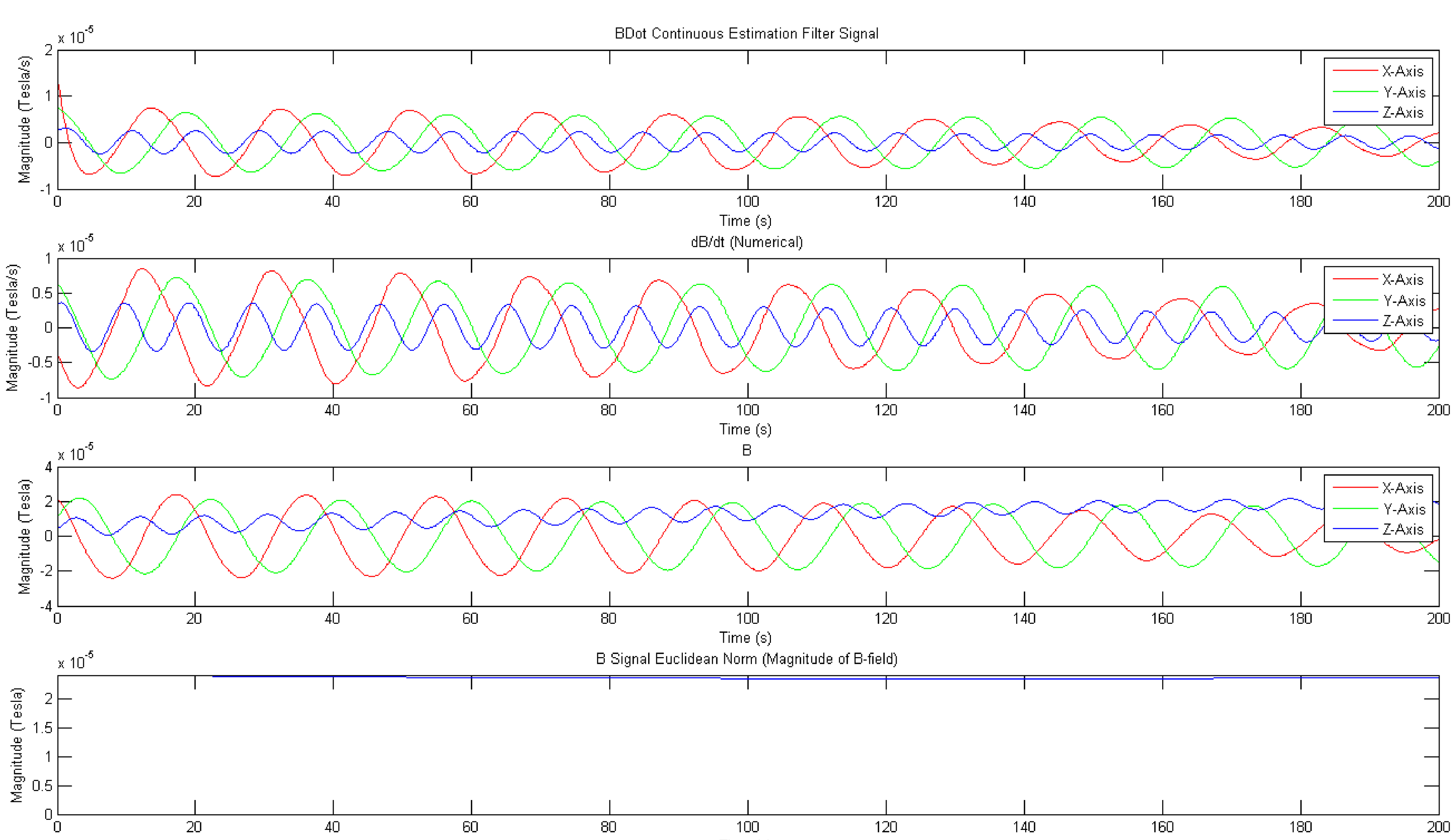
$$a_{drag} = -\frac{1}{2}\rho_a v^2 \frac{C_d A}{m}$$

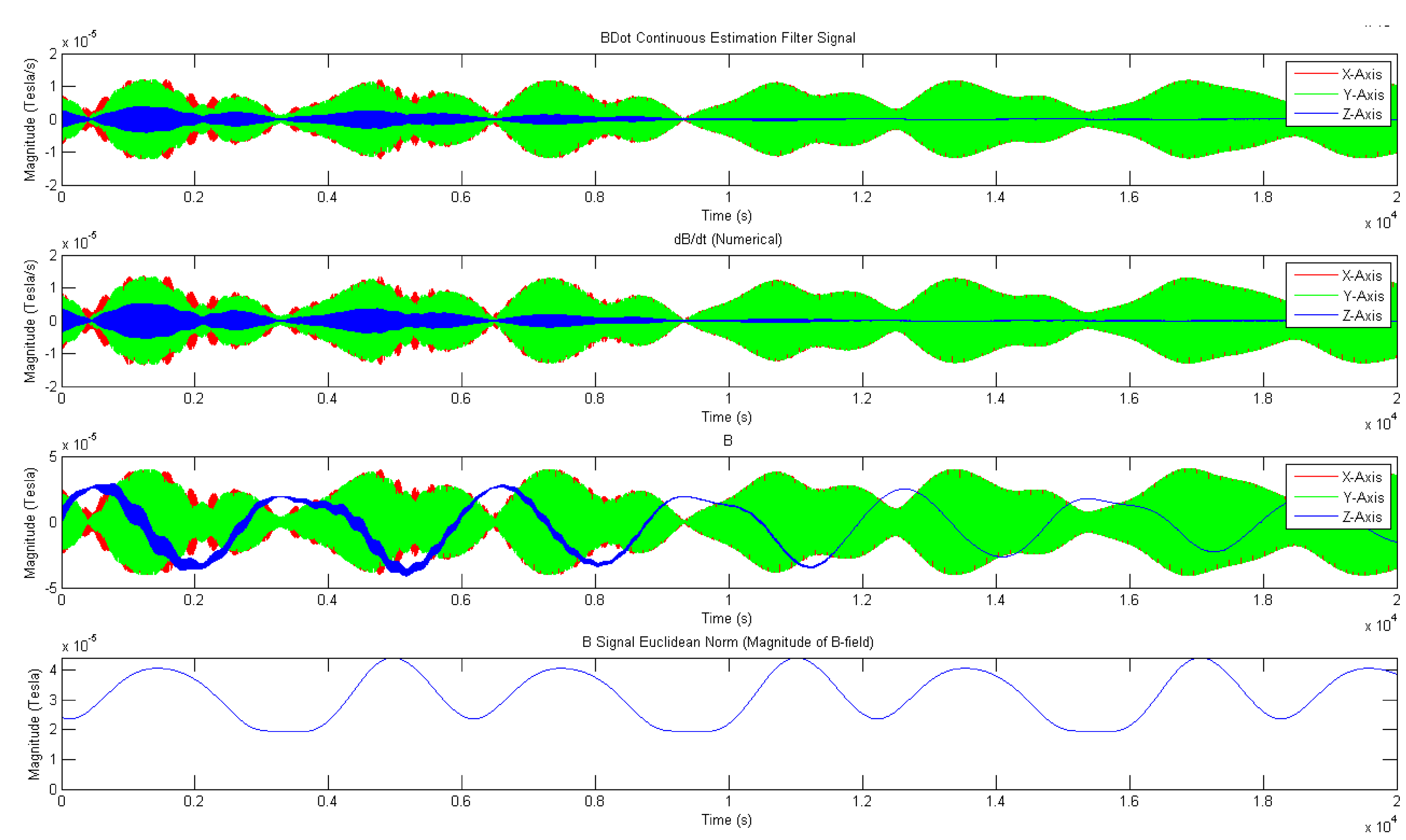
Propagating Attitude State

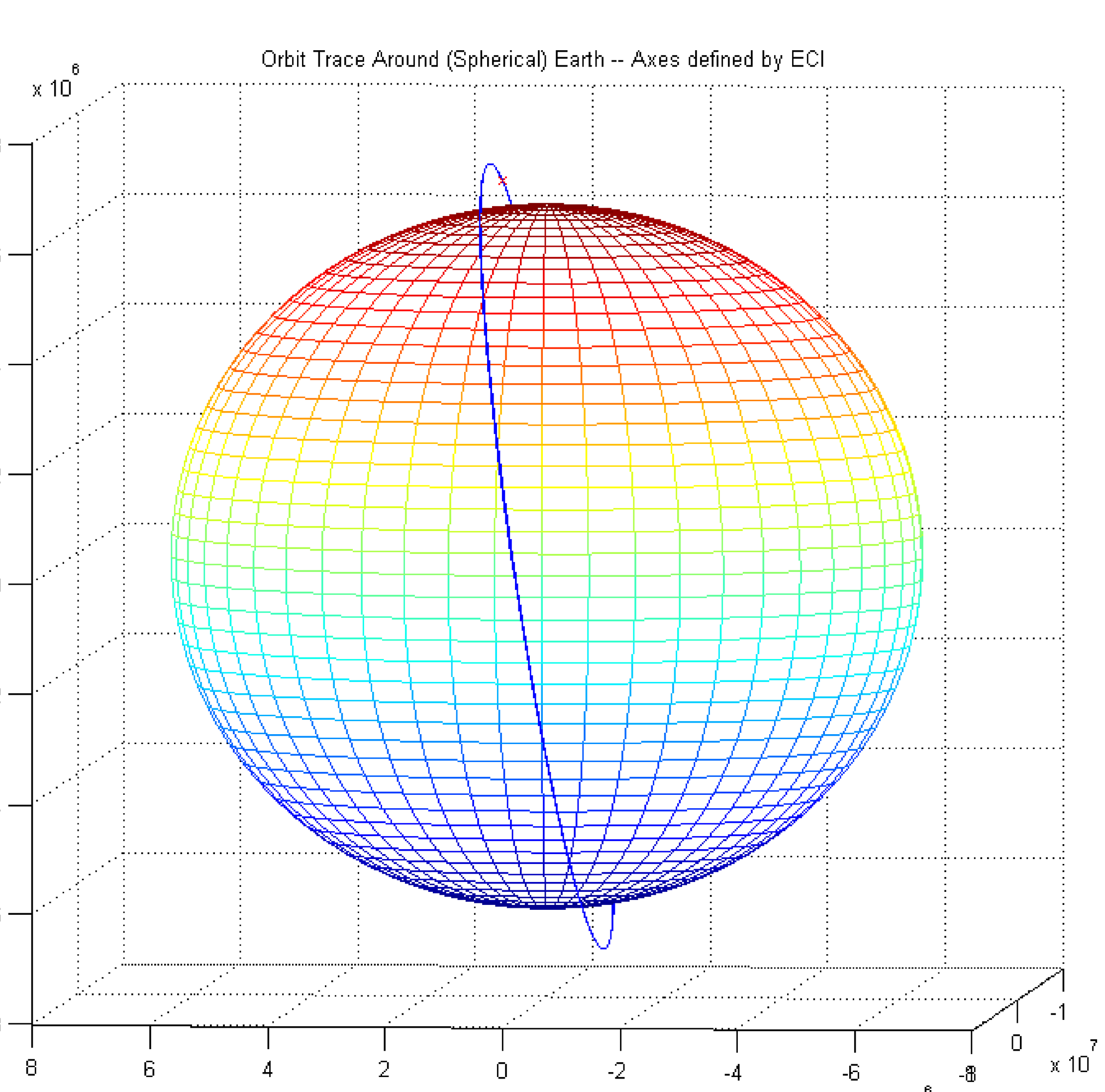
Euler's equations of rigid body dynamics are used to simulate the rotation of the satellite. When coupled with quaternion mathematics, the differential changes in the quaternion vector and the angular velocity vector are given by the following equations (Coutsias, 14-15), where *q* is the quaternion vector, *Ω* is the angular velocity vector, *I* is the vector containing the principal moments of inertia, and *T* is the vector of external torques:

$$\frac{dq}{dt} = \frac{1}{2}q\Omega = \frac{1}{2}\begin{pmatrix} 0 & -\Omega_1 & -\Omega_2 & -\Omega_3 \\ \Omega_1 & 0 & \Omega_3 & -\Omega_2 \\ \Omega_2 & -\Omega_3 & 0 & \Omega_1 \\ \Omega_3 & \Omega_2 & -\Omega_1 & 0 \end{pmatrix}\begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \begin{pmatrix} \frac{I_2 - I_3}{I_1}\Omega_2\Omega_3 + \frac{T_1}{I_1} \\ \frac{I_3 - I_1}{I_2}\Omega_3\Omega_1 + \frac{T_2}{I_2} \\ \frac{I_1 - I_2}{I_3}\Omega_1\Omega_2 + \frac{T_3}{I_3} \end{pmatrix}$$







Example Results of Simulation

[1, middle left] Plot of magnetic field vector in spacecraft body coordinates, as would be "sensed" by on-board magnetometers. These sinusoids are produced as a result of rotation of the satellite (no external torques). The magnitude of the magnetic field remains relatively constant over a few spins, but drops a small amount due to orbit propagation.

[2, middle right] Identical plot to [1], showing the fluctuations that occur over an entire orbit. This plot is also the result of a detumble algorithm running in the control section of the simulation, and the "thinning" of the B field in the Z component can be attributed to the angular velocities in X and Y being transferred to solely Z.

[3, bottom left] Example of a trace of the orbit path as a result of the simulation. Orbit parameters can be varied to produce desired results.

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Credits & References

- **Sennan Sulaiman** (previous ELFIN ADCS lead) for establishing the simulation framework (and the rest of the ELFIN staff and student team for their work)

- **B. Young & DANDE Mission** for providing a basis framework for propagating the orbit and attitude of a spacecraft

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